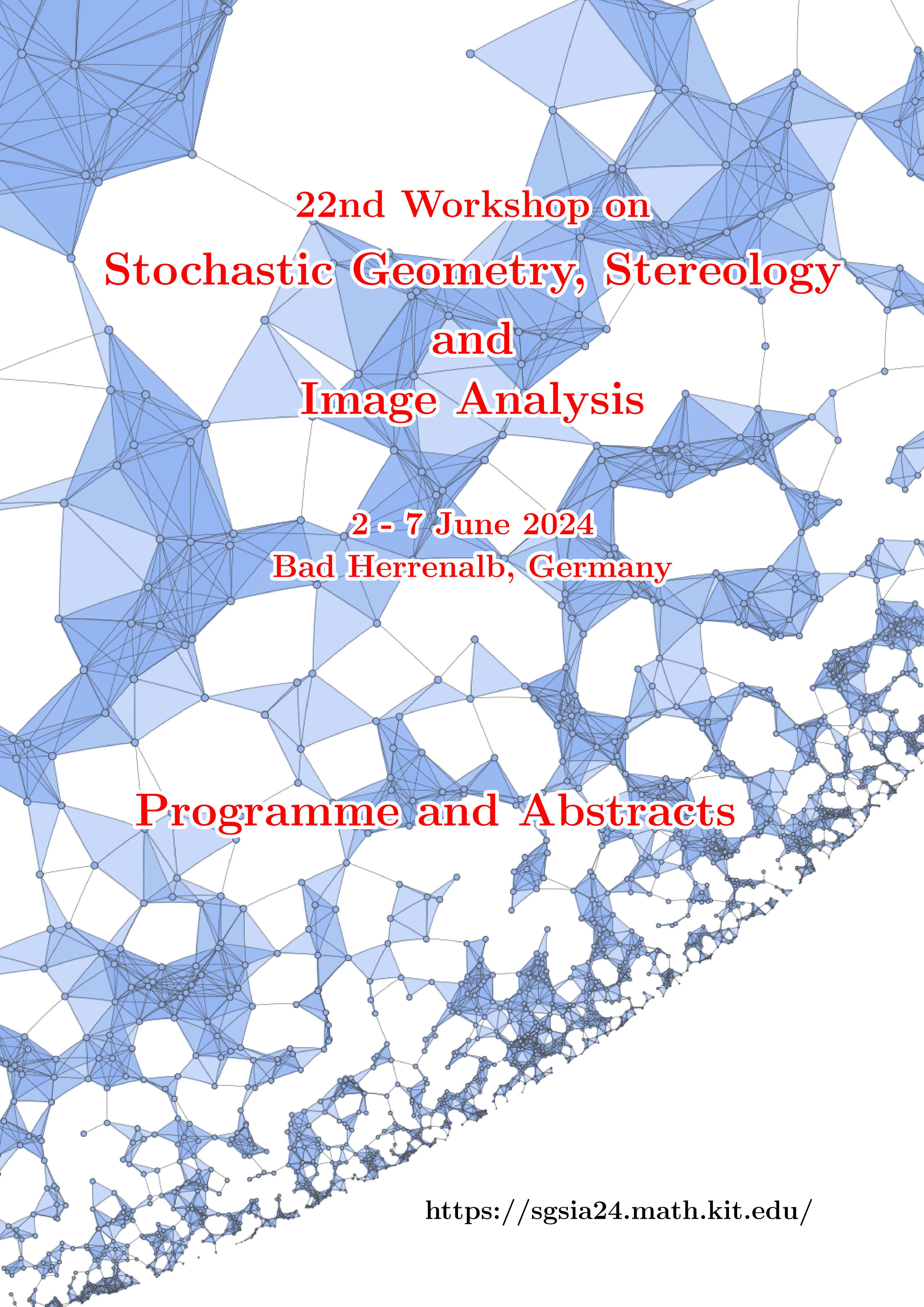

22nd Workshop on

Stochastic Geometry, Stereology and Image Analysis

2 - 7 June 2024

Bad Herrenalb, Germany

Programme and Abstracts

https://sgsia24.math.kit.edu/

Conference Venue

Our conference venue **Haus der Kirche** is a conference center of the Protestant Church of Germany. It is situated in *Bad Herrenalb*, a health resort in the northern part of the Black Forest. The town centre with its spa gardens, mineral springs and some remains of a medieval monastery as well as the train station are all within walking distance from the conference venue. The street address is *Evangelische Akademie Baden, Dobler Str. 51, 76332 Bad Herrenalb, Germany.*

For a street map of Bad Herrenalb see the inside of the front cover.

Meals, Drinks and Conference Dinner

Breakfast is served from 7.30 to 9.00 am.

Lunch is served at 12.30 noon and **dinner** at 6.30 pm.

During **coffee breaks** water, coffee and tea will be available. Water will also be available during all meals. Other drinks are available at any time but are not covered. There are several fridges and shelves with drinks throughout the house with an honesty box beside them. It is also possible to get a drinks checklist from the reception desk in which you can mark your consumed drinks during the week and pay when you check out.

If you have any special **dietary requirements** or food allergies, please discuss these directly with the hotel staff or let us know.

On Thursday evening you are cordially invited to a **conference dinner barbecue** starting at 6.30 pm to celebrate a hopefully inspiring and enjoyable conference.

Wireless Internet Access

Wireless internet access is provided free of charge for all participants within the conference venue. There is an *open* network *BADEN-WLAN*, which you can access directly without any login. And there is the *secure* network *BADEN-sWLAN*, for which you need to register as a user. Upon registration you will receive an e-mail with a confirmation code and your login credentials.

Session Format and Talk Style

For the sake of lively discussion, please respect the following maximal talk times allocated to your presentations:

Keynote lectures: 40 minutes talk time;
Contributed talks: 30 minutes talk time.

The lecture hall is equipped with a computer and a video projector. There is also some black or white board available. We recommend to prepare slides as the board may be too small to rely on it for a whole talk. Please upload the slides of your talk (preferably in pdf-format) to the computer in the lecture room well before the start of the session allocated to your presentation. It is also possible to send the file by email to sgsia24@math.kit.edu (preferably on the day before your session).

Poster Session

Posters are displayed for the duration of the conference.

There is a dedicated **Poster Presentation Session** on Monday before dinner starting at 17:40 h, where each poster is introduced in a 2 minutes short presentation. After dinner it is continued with a **Poster Session**, which allows for individual and more in-depth discussions. Abstracts of the posters can be found in this booklet starting from page 35.

Monday, 3 June 2024

09:00 – 09:10		**Welcome**
09:10 – 10:00	**KN Müller**	*Poisson-Voronoi and Poisson-Gabriel percolation, in high dimensions*
10:00 – 10:40	**Yukich**	*Limit theory for dynamic spatial random models*
coffee break		
11:10 – 11:50	**Trauthwein**	*CLTs for Poisson-based Random Graphs*
11:50 – 12:30	**Pabst**	*Sharp phase transition for percolation in the RCM for higher-dimensional simplicial complexes*
lunch break		
14:30 – 15:10	**Trapp**	*Lower variance bounds for Poisson functionals with an application to Poisson shot noise processes*
15:10 – 15:50	**Penrose**	*Coverage from within in the Boolean and Johnson-Mehl models*
coffee break		
16:20 – 17:00	**Kiderlen**	*Confidence Intervals for Newton-Cotes Quadratures Based on Stationary Point Processes*
17:00 – 17:40	**Staněk**	*Multivariate asymptotic test of pairwise independence for orientations with the same symmetry group*
17:40 – 18:20		**Poster Presentations**
dinner		
19:30 – 21:00		**Poster Session**

Keynote lecture	Tobias Müller	09:10 – 10:00

Poisson-Voronoi and Poisson-Gabriel percolation, in high dimensions

Tobias Müller
(Groningen University)

We will consider percolation on the Voronoi tessellation generated by a homogeneous Poisson point process on d-dimensional space, and on the Gabriel graph defined on such a process. Each cell of the Voronoi tessellation, respectively each point of the graph, is colored black with probability p and white with probability $1 - p$ (independently of all other cells/points). Percolation occurs if there exists an infinite, black connected cluster of cells/points, and the critical probability p_c is defined as the infimum of all values p for which the probability of percolation is positive. Famously, it was shown by Bollobas and Riordan in 2005 that $p_c = 1/2$ for Voronoi percolation in dimension $d = 2$. In the talk I plan to discuss a proof that, as d tends to infinity, we have $p_c = (1 + o(1))(e/d)2^{-d}$ for Poisson-Voronoi percolation and $p_c = (1 + o(1))2^{-d}$ for Poisson-Gabriel percolation.

Based on joint work with Rene Conijn, Matthias Irlbeck and Zakhar Kabluchko.

Talk	Joseph Yukich	10:00 – 10:40

Limit theory for dynamic spatial random models

Joseph Yukich
(Lehigh University)

We establish the limit theory for statistics of spatial random models evolving over a time domain and which are asymptotically de-correlated over spatial domains. The sources of model randomness given by the random collection of particle locations, their random initial states, and the system evolution, give rise to point processes with interacting time-evolving marks. We control the model randomness via weak localization conditions on the particle locations as well as a localization condition on the marks allowing for unbounded interactions between marks. In this way we establish the limit theory for statistics of the model as the spatial domain increases up to $\mathbb{R}^d$. This gives weak laws of large numbers, variance asymptotics, and asymptotic normality for statistics of spatial random models having unbounded interactions, including continuum versions of spin models and interacting diffusion models.

The talk is based on joint work with B. Blaszczyszyn and D. Yogeshwaran.

Talk	Tara Trauthwein	11:10 – 11:50

CLTs for Poisson-based Random Graphs

Tara Trauthwein
(University of Oxford)

In this talk, I will show how to derive Central Limit Theorems for a certain type of functions of Poisson measures, under minimal moment assumptions and using the Malliavin-Stein method. Special attention will be given to applications in the field of random graphs.

Talk	Dominik Pabst	11:50 – 12:30

Sharp phase transition for percolation in the RCM for higher-dimensional simplicial complexes

Dominik Pabst
(Karlsruhe Institute of Technology)

Random simplicial complexes have become very popular in the literature during the last years. In this talk we will present a new model of random simplicial complexes based on the Random Connection Model (RCM). We consider percolation in this random simplicial complex based on the notion of up connectivity, a generalisation of the usual connectivity for graphs to simplicial complexes. The focus of the talk will be on a sharp phase transition for percolation in this model under some conditions on the so-called connection functions. A similar result is already known for the Čech and the Vietoris-Rips complex, which appear as special cases of the model considered in this talk.

Talk	Vanessa Trapp	14:30 – 15:10

Lower variance bounds for Poisson functionals with an application to Poisson shot noise processes

Vanessa Trapp
(Hamburg University of Technology)

Lower bounds for variances are often needed to derive central limit theorems. In this talk, a generalised reverse Poincaré inequality is introduced, which provides a lower variance bound for Poisson functionals that depends on the difference operator of some fixed order. To show how to apply this lower variance bound we consider a class of Poisson shot noise processes whose excursion sets within compact convex observation windows are almost surely polyconvex. In this talk, we analyse the behaviour of specific geometric functionals of these excursion sets for growing observation windows, derive a lower variance bound and show a central limit theorem.

Talk	Mathew Penrose	15:10 – 15:50

Coverage from within in the Boolean and Johnson-Mehl models

Mathew Penrose
(University of Bath)

Given a compact planar region A with a nice boundary, let τ_A be the (random) time it takes for A to be fully covered by a spatial birth-growth process in A with seeds arriving as a unit-intensity Poisson point process in $A \times [0, \infty)$, where upon arrival each seed grows at unit rate in all directions. We show that for any real x, in the large-s limit we have

$$\Pr[\pi\tau_{sA}^3 - 6\log s - 4\lg\lg s \leq x] \to \exp\left(-(\frac{81}{4\pi})^{1/3}|A|e^{-x/3} - (\frac{9}{2\pi^2})^{1/3}|\partial A|e^{-x/6}\right).$$

The second term in the exponent is due to boundary effects, the importance of which was not recognized in earlier work on this model. We present similar results in higher dimensions (where boundary effects dominate). These are derived using new results on the asymptotic probability of covering A with a high-intensity spherical Poisson Boolean model *restricted to* A with grains having iid small random radii satisfying a near-optimal moment condition.

The corresponding result for coverage of A by an *unrestricted* Boolean model is a classic result of Hall and of Janson, who did not give any error bounds. We provide an $O(1/\log n)$ rate of convergence in their result.

This is joint work with Frankie Higgs and with Xiaochuan Yang, supported by EPSRC Grant EP/T028653/1.

Talk	Markus Kiderlen	16:20 – 17:00

Confidence Intervals for Newton-Cotes Quadratures Based on Stationary Point Processes

Markus Kiderlen
(Aarhus University)

Motivated by the stereological application of volume estimation, we consider the problem of numerical integration on the real line, employing function values at a finite set of randomly chosen points. The sampling points are modeled by a stationary point process, with the estimators being Newton-Cotes quadratures. Our comprehensive probabilistic analysis crucially extends existing results regarding the approximation error and variance, accommodating even non-ergodic sampling processes. Notably, these findings are used to formulate novel asymptotic confidence intervals, a considerable challenge given the usual absence of limit distributions.

To underscore the practicality of our approach, we apply it to a stereological simulation study. Specifically, we establish confidence intervals for the volume of a three-dimensional ellipsoid, based on section areas obtained from randomly positioned parallel planes.

Joint work with Mads Stehr.

Talk	Jakub Staněk	17:00 – 17:40

Multivariate asymptotic test of pairwise independence for orientations with the same symmetry group

Jakub Staněk
(Charles University)

The contribution concerns the independence of r-tuples of orientations of symmetrical objects in three-dimensional space. Beyond its applications in crystallography and material science, such objects occur in various fields. The characteristics of random orientations and the properties of their estimators are discussed. The main contribution is constructing multivariate asymptotic tests of pairwise independence based on the theory of U-statistics and a novel definition of covariance between two orientations. The finite-sample performance of the tests is assessed through a simulation study, investigating their power on three models for r-tuples of orientations and comparing them to a permutation test. The application of the test is demonstrated on a dataset of polycrystalline material with cubic symmetry of the crystal lattice.

Joint work with Iva Karafiátová and Zbyněk Pawlas.

Tuesday, 4 June 2024

09:00 – 09:50	**KN Redenbach**	*Image segmentation by neural networks trained on synthetic data*
09:50 – 10:30	**Nguyen**	*A statistical method for crack detection in 3D concrete images*
coffee break		
11:00 – 11:40	**Neumann**	*Stochastic 3D modeling of the nanoporous binder-additive phase in battery electrodes*
11:40 – 12:20	**Gotovac Đogaš**	*Comparison of random sets distributions via statistical depths*
lunch break		
14:30 – 15:20	**KN Lederer**	*Sparsity in Data Science: Selected Trends*
15:20 – 16:00	**Zuyev**	*Training Bayesian neural networks with measure optimisation algorithms*
coffee break		
16:30 – 17:10	**Korfhage**	*Existence of a robust phase in the Poisson Boolean model with convex grains*
17:10 – 17:50	**Ilienko**	*Small-scale asymptotic structure of ordered uniform k-spacings*
17:50 – 18:30	**Svane**	*Normal approximation for Gibbs processes via disagreement coupling*
dinner		

Keynote lecture	Claudia Redenbach	09:00 – 09:50

Image segmentation by neural networks trained on synthetic data

Claudia Redenbach
(RPTU Kaiserslautern-Landau)

Neural networks are commonly used for image segmentation. Training a network requires an appropriate amount of training data, that is, images along with the desired segmentation result. Manual annotation of images is common practice, but time-consuming and error-prone. We propose the use of synthetic image data. For their simulation, virtual micro structures are generated from stochastic geometry models. A model of the imaging process is then applied to simulate realistic images of the synthetic structures. Binary images of the model realizations provide a ground truth for the segmentation. The resulting pairs are then used to train the neural network. We present two application examples: segmentation of cracks in computed tomography images of concrete and of FIB-SEM images of porous structures. To support the selection of training data and the evaluation of the segmentation results, we introduce similarity measures for geometric microstructure models and binary discretizations of random sets.

Talk	Duc Nguyen	09:50 – 10:30

A statistical method for crack detection in 3D concrete images

Duc Nguyen
(Ulm University)

In practical applications, effectively segmenting cracks in large-scale computed tomography (CT) images holds significant importance for understanding the structural integrity of materials. However, classical methods and Machine Learning algorithms often incur high computational costs when dealing with the substantial size of input images. Hence, a robust algorithm is needed to predetect crack regions, enabling focused analysis and reducing computational overhead. The proposed approach addresses this challenge by offering a streamlined method for identifying crack regions in CT images with high probability. By efficiently identifying areas of interest, our algorithm allows for a more focused examination of potential anomalies within the material structure. Through comprehensive testing on both semi-synthetic and real 3D CT images, we validate the efficiency of our approach in enhancing crack segmentation while reducing computational resource requirements.

Joint work with Vitalii Makogin and Evgeny Spodarev.

Talk	Matthias Neumann	11:00 – 11:40

Stochastic 3D modeling of the nanoporous binder-additive phase in battery electrodes

Matthias Neumann
(Graz University of Technology)

A stochastic 3D model for the nanoporous binder-additive phase in hierarchically structured electrodes of lithium-ion batteries is presented. The considered binder-additive phase consists of carbon black, polyvinylidene difluoride binder and graphite particles. For stochastic 3D modeling, we use a three-step approach combining excursion sets of Gaussian random fields with germ-grain models. First, the graphite particles extracted from image data are modeled by a Boolean model with ellipsoidal grains. Second, the union of carbon black and binder is modeled by an excursion set of a Gaussian random field in the complement of the graphite particles. Third, large pore regions within the union of carbon black and binder are modeled by a Boolean model with spherical grains. The model is calibrated to tomographic image data of cathodes in lithium-ion batteries acquired by focused ion beam scanning electron microscopy (FIB-SEM). Subsequently, model validation is performed by comparing various morphological descriptors, that are not used for model fitting, of both model realizations and measured image data. Moreover, we use the validated model for generating virtual, yet realistic, image data of the nanoporous binder-additive phase with systematic variations in the volume fraction of graphite particles. The latter can be controlled by adjusting the intensity of the Boolean model with ellipsoidal grains. Based on this analysis, we quantitatively study the influence of graphite particles on morphological descriptors as well as on effective transport properties such as effective conductivity in the binder-additive phase and effective diffusivity in the pore space.

Joint work with Phillip Gräfensteiner and Volker Schmidt.

Talk	Vesna Gotovac Đogaš	11:40 – 12:20

Comparison of random sets distributions via statistical depths

Vesna Gotovac Đogaš
(University of Split)

We present several depths for possibly non-convex random sets. The depths are applied to the comparison between two samples of non-convex random sets, using a visual method of DD-plots and statistical tests. The advantage of this approach is to identify sets within the sample that are responsible for rejecting the null hypothesis of equality of the distribution and to provide clues on differences between the distributions. The method is justified on the basis of a simulation study.

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[2] Møller J., Waagepetersen R.P. (2004): *Statistical Inference and Simulation for Spatial Point Processes.* Chapman and Hall/CRC, Boca Raton.

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[5] Liu R. Y., Parelius. J. M., Signh K. (1999): Multivariate analysis by data depth: Descriptive statistics, graphics and inference, *The Annals of Statistics* **27**(3), 783–858.

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[7] Whitaker R.T., Mirzargar M., Kirby R.M. (2013): Contour boxplots: a method for characterizing uncertainty in feature sets from simulation ensembles *IEEE Transactions on Visualization and Computer Graphics* 19, 2713–2722.

Keynote lecture	Johannes Lederer	14:30 – 15:20

Sparsity in Data Science: Selected Trends

Johannes Lederer
(University of Hamburg)

Sparsity can avoid overfitting, speed up computations, and facilitate interpretations. This presentation recaps sparsity in the framework of "classical" high-dimensional statistics. It then introduces corresponding notions in modern data-science frameworks, such as deep learning and high-dimensional extremes. Along the way, we discuss different perspectives on data science and establish connections between these perspectives.

Talk	Sergei Zuyev	15:20 – 16:00

Training Bayesian neural networks with measure optimisation algorithms

Sergei Zuyev
(Chalmers University of Technology)

On a high abstraction level, a Bayesian neural network (BNN) can be seen as a function of input data and their prior probability distribution which yields, among other outputs, their estimated posterior probability distribution. This distribution is a result of optimisation of a chosen score function aiming to favour these probability distributions which describe best the observed data and take into account the prior distribution.

Instead of constraint optimisation over the simplex of probability distributions, it is typical to map this simplex into Euclidean space, for example with Softmax function or its variants, and then do optimisation in the whole space without constraints. It is, however, widely acknowledged that such mapping often suffers from undesirable properties for optimisation and stability of the algorithms. To counterfeit this, a few regularisation procedures have been proposed in the literature.

Instead of trying to modify the mapping approach, we suggest turning back to optimisation on the original simplex space using recently developed algorithms for constrained optimisation of functionals of measures. We demonstrate that our algorithms run tens times faster than the standard algorithms involving softmax mapping and lead to exact solutions rather than to their approximations.

Talk	**Marilyn Korfhage**	**16:30 – 17:10**

Existence of a robust phase in the Poisson Boolean model with convex grains

Marilyn Korfhage
(University of Cologne)

Consider a homogenous Poisson Point Process on $\mathbb{R}^d$, equipped with *convex grains*, i.e. i.i.d. copies of a random convex body that is rotationally-invariant in distribution. We define for such a convex body a non-increasing sequence of diameters. The first diameter is the classical diameter of the convex body. The i-th diameter is then defined as the diameter of the orthogonal projection of the body from the previous step along the $(i-1)$-st diameter onto the $(d-i+1)$-dimensional hyperplane. This projected body is then projected further when determining the next diameter. We state several criteria on the diameter distribution and moment conditions for the volume of the convex body that result in either a dense process, i.e. the whole space is covered by the grains, or robustness, i.e. the union of the grains has an unbounded connected component for any intensity of the underlying Poisson process. Importantly, we do not impose any conditions on the joint distribution of the diameters. If the grains are chosen to be euclidean balls, it is known that density and robustness are equivalent. We show in our general model that in any dimension $d \geq 2$ there exists grain distributions where robustness does not imply density.

Joint work with Peter Gracar and Peter Mörters.

Talk	Andrii Ilienko	17:10 – 17:50

Small-scale asymptotic structure of ordered uniform k-spacings

Andrii Ilienko
(University of Bern and National Technical University of Ukraine)

As is well known, uniform spacings are constructed as follows. Consider $n - 1$ independent points uniformly distributed across the unit interval. Uniform 1-spacings are defined as the distances between adjacent points, while k-spacings are the distances between points that have exactly $k - 1$ other points between them. There is a substantial literature on the exact and asymptotic results concerning uniform 1-spacings (and to a lesser extent, k-spacings) and their numerous statistical applications.

We propose an apparently novel concept of local Poisson approximation for k-spacings, which enables gaining detailed insights into their asymptotic behavior at small scales across the entire range of their possible values. This concept not only facilitates the straightforward proof of known limit theorems for minimum and maximum k-spacings, but also allows for the study of the asymptotic properties of all k-spacings, regardless of length.

Talk	Anne Marie Svane	17:50 – 18:30

Normal approximation for Gibbs processes via disagreement coupling

Anne Marie Svane
(Aalborg University)

We consider normal approximation of geometric functionals of finite-volume Gibbs point processes in increasing windows. Most existing results rely on a perfect simulation coupling available only for processes with very limited interaction range. To improve on the class of Gibbs processes, we apply instead the recently developed disagreement coupling technique. This results in two central limit theorems: a qualitative CLT for weakly stabilizing functionals and a quantitative CLT for exponentially stabilizing functionals. The first result is shown via martingale methods, while the second proof makes use of Stein couplings. Examples of functionals verifying the conditions in the weakly stabilizing case are persistent Betti number and total edge length of Euclidean minimal spanning trees, and examples in the exponentially stabilizing case are k-nearest neighbor distances and total edge length of Voronoi cells.

This is joint work with C. Hirsch and M. Otto.

Wednesday, 5 June 2024

09:00 – 09:50	**KN Hirsch**	*Large deviations for geometric functionals in Euclidean and hyperbolic space*
09:50 – 10:30	**Heerten**	*Cumulant Method for Weighted Random Connection Models*
coffee break		
11:00 – 11:40	**Privault**	*Normal approximation of subgraph counts in the random-connection model*
11:40 – 12:20	**Bhattacharjee**	*Spectra of Poisson functionals and applications in continuum percolation*
lunch break		
13:30 – 18:30	**Excursions/ Free time**	
dinner		

Keynote lecture	Christian Hirsch	09:00 – 09:50

Large deviations for geometric functionals in Euclidean and hyperbolic space

Christian Hirsch
(Aarhus University)

In this talk, I will discuss large deviation principles (LDPs) for geometric functionals on Poisson point processes in increasing windows. The guiding examples are k-nearest neighbor distances and persistent Betti numbers. These LDPs are of level two, i.e., they are on the level of random measures. The results cover both the sparse and the dense scaling limit of the connectivity radius.

In the sparse case, I will consider the point process associated with k-element connected components in $\mathbb{R}^d$ as the connectivity radii tend to 0. The rate function for the obtained LDP can be represented as relative entropy.

In contrast, in the dense case, my attention will be on k-nearest neighbor balls exceeding a certain volume threshold. I'll demonstrate the LDP under the condition that the growth rate of centering terms for the volume of k-nearest neighbor balls is slower than that required for Poisson convergence.

Lastly, I will explore extensions of these concepts to hyperbolic space.

This talk is based on joint work with T. Kang, M. Otto, T. Owada, and Ch. Thäle.

Talk	Nils Heerten	09:50 – 10:30

Cumulant Method for Weighted Random Connection Models

Nils Heerten
(Ruhr University Bochum)

We derive cumulant bounds for subgraph counts and power-weighted edge lengths in a class of spatial random networks known as weighted random connection models. By the method of cumulants, these bounds lead to a variety of asymptitic results for this statistics, of which a moderate deviation principle, normal approximation with Cramér corrections and a Bernstein-type concentration inequality are discussed here. Deriving the bounds involves dealing with long-range spatial correlations induced by the profile function and the weight distribution. In this talk, we mainly focus on the classical case of a Poisson vertex set, but also provide extensions to α-determinantal processes.

This is a joint work with Christian Hirsch and Moritz Otto (arXiv:2311.00600).

Talk	Nicolas Privault	11:00 – 11:40

Normal approximation of subgraph counts in the random-connection model

Nicolas Privault
(Nanyang Technological University)

We derive normal approximation results for subgraph counts written as multiparameter stochastic integrals in a random-connection model based on a Poisson point process. By combinatorial arguments we express the cumulants of general subgraph counts using sums over connected partition diagrams, after cancellation of terms obtained by Möbius inversion. Using the Statulevičius condition, we deduce convergence rates in the Kolmogorov distance by studying the growth of subgraph count cumulants as the intensity of the underlying Poisson point process tends to infinity. Our analysis covers dilute and sparse random graph regimes, and can take into account an arbitrary configuration of fixed subgraph vertices. Numerical examples are presented using a computer code in SageMath for the closed-form computation of cumulants of any order, for any type of connected subgraph and for any configuration of endpoints in any dimension $d \geq 1$. In particular, graph connectivity estimates, Gram-Charlier expansions for density estimation, and correlation estimates for joint subgraph counting are obtained.

References

LP23 Q. Liu and N. Privault. Graph connectivity with fixed endpoints in the random-connection model. *Preprint arXiv:2312.12745*, 2023.

LP24 Q. Liu and N. Privault. Normal approximation of subgraph counts in the random-connection model, to appear in Bernoulli, 2024.

Talk **Chinmoy Bhattacharjee** **11:40 – 12:20**

Spectra of Poisson functionals and applications in continuum percolation

Chinmoy Bhattacharjee
(Universität Hamburg)

In this talk, I will consider the noise sensitivity of dynamical critical planer continuum percolation models, such as the Boolean model and Voronoi percolation model. While similar results for the Voronoi percolation have previously been shown in Vanneuville (2021) under the so-called frozen dynamics, we instead consider the Ornstein-Uhlenbeck (OU) dynamics.

A critical planer dynamical percolation model is said to be noise sensitive if the ± 1-indicator of a left-right occupied crossing of large squares of side length L in the model is sensitive to small noises in the underlying system. We introduce the noise according to the OU dynamics and show a sharp transition result : when the amount of noise tends to zero as $L \to \infty$ fast enough, then the model is not sensitive to the noise, while if it doesn't tend to zero fast enough, the model becomes noise sensitive. The main tool is a notion of spectral point process based on the chaos expansion of the crossing functionals, which parallels the corresponding notion of spectral samples in the discrete setting.

The talk is based of a joint work with Giovanni Peccati and Yogeshwaran Dhandapani.

Thursday, 6 June 2024

09:00 – 09:50	**KN Klatt**	*Hyperuniformity: Why? How? What?*
09:50 – 10:30	**Mastrilli**	*Estimating the structure factor exponent at zero-wavelength for hyperuniform point processes*
coffee break		
11:00 – 11:40	**Lachiéze-Rey**	*Optimal transport and matching of point processes*
11:40 – 12:20	**Wittmann**	*The mathematics of classical density functional theory*
lunch break		
14:30 – 15:20	**KN Gusakova**	*Poisson hyperplane tessellation in constant curvature spaces*
15:20 – 16:00	**Hug**	*Boolean Models in Hyperbolic Space*
coffee break		
16:30 – 17:10	**Calka**	*Fractal random sets generated by iterated planar regular tessellations*
17:10 – 17:50	**Gommes**	*Time-dependent stochastic models for inelastic neutron scattering data analysis*
17:50 – 18:30	**Molchanov**	*Set-valued recursions arising from vantage-point trees*
20:00	**conference dinner**	

Keynote lecture **Michael Andreas Klatt** **09:00 – 09:50**

Hyperuniformity: Why? How? What?

Michael Andreas Klatt
(German Aerospace Center (DLR))

A hyperuniform system can be isotropic and locally disorderd, like a liquid, but at large scales it becomes homogeneous, like a crystal. This combination endows hyperuniform materials with unique physical properties. At the same time, the concept provides interdisciplinary challenges. This talk presents an overview of at least some recent progress from physics and mathematics, e.g., on how to detect hyperuniformity, what fluctuations preserve or destroy hyperuniformity, and what might distinguish different types of hyperuniform disorder in the thermodynamic limit.

Talk **Gabriel Mastrilli** **09:50 – 10:30**

Estimating the structure factor exponent at zero-wavelength for hyperuniform point processes

Gabriel Mastrilli
(ENSAI Rennes)

A point process Φ is hyperuniform if the variance of $\#\{\Phi \cap B(0,R)\}/|B(0,R)|$ tends to zero as R approaches infinity. Initially conceptualized in statistical physics, hyperuniformity has gained attention in various applied contexts. Despite this, statistical tests for hyperuniformity have only recently gained attention. The notion of hyperuniformity can be redefined using the structure factor S of Φ (that correspond to the density of the Bartlett's spectral measure). To be more precise, under certain conditions on the process, hyperuniformity becomes equivalent to the structure factor $S(k)$ vanishing at zero frequency $k = 0$. This approach allows also for the classification of processes based on how quickly their structure factor S diminishes at zero. A common assumption is that S follows a power-law behavior near zero: $S(k) \sim |k|^\alpha$ as $|k| \to 0$. Accordingly, statistical estimation of α may face the challenge to estimate the structure factor S for small frequencies.

In this talk, we will construct a family of estimators for α, that shortcut the step of estimating $S(k)$ for small but non zero frequencies k, to estimate α at $S(0)$. Then, after having discussing their convergence properties, we will explain how to ensure a valuable estimation of α with just one realization of the point process.

Joint work with Bartłomiej Błaszczyszyn and Frédéric Lavancier.

Talk	Raphaël Lachiéze-Rey	11:00 – 11:40

Optimal transport and matching of point processes

Raphaël Lachiéze-Rey
(INRIA Paris and Université Paris Cité)

A matching between two point processes is a one-to-one mapping. We investigate the properties of optimal matchings between two stationary point processes with unit intensity, aiming at minimising the distance between two typically matched points, constituting a specific instance of the problem of optimal transport. We show that under mild conditions, a stationary point process behaves as a Poisson process in terms of magnitude. We also investigate the case of Hyperuniform point processes, encompassing many models of random matrices and Coulomb gases. These processes are characterized by low variance in the number of points within a large window, and despite their locally disordered structure, they are anticipated to exhibit global behavior akin to perturbed lattices. We prove that their expected transport cost is indeed comparable to that of a perturbed lattice, which yields in particular that in dimension 1 and 2 this cost is negligible with respect to that of disordered processes such as Poisson processes.

Joint work with Yogeshwaran D.

Talk	René Wittmann	11:40 – 12:20

The mathematics of classical density functional theory

René Wittmann
(Max Rubner-Institut Kulmbach and Heinrich-Heine-Universität Düsseldorf)

Classical density functional theory (DFT) is a versatile tool in Statistical Physics to describe the structure and phase behavior of an interacting fluid in thermal equilibrium [1], i.e., an inhomogeneous marked Gibbs process. This formally exact approach builds upon a rigorously proven uniqueness theorem for the density-potential mapping [2], which basically implies that the density (or intensity) of the particle coordinates in space, which follows from a variational principle in DFT, contains all information on the system. In practical applications of DFT to most physical systems, approximations are needed.

For anisotropic (convex) particles with hard-core excluded-volume interactions, fundamental-measure based DFT provides an accurate and elegant way to predict the fluid properties from the particle shape alone. This is because of its deep foundation in differential and integral geometry. Recognizing that the Mayer function, which represents the interaction between two hard particles, corresponds to the Euler characteristic of the intersection of two convex bodies, it can be decomposed in terms of local valuations (curvature measures, tensors, or mixed measures) to provide the central building blocks of fundamental-measure DFT.

In this presentation, I give a general introduction to the versatile framework of DFT and the underlying mathematical concepts, before presenting two recent applications to colloidal liquid crystals. First, I derive an analytic scheme to predict the onset of phase biaxiality [3], i.e., when there exist two different directions of preferred orientational alignment. Second, I describe the fine structure of topological defects in geometrically frustrated two-dimensional smectics, i.e., layered systems in which the rod-like particles want to align in parallel layers, where DFT reproduces detailed observations from experiments and numerical simulations [4]. I conclude by discussing rigorous results, which establish in how far the underlying uniqueness theorem of DFT extends to time-dependent density-potential mappings [5].

Joint work with Anouar El Moumane, Paul A. Monderkamp, Michael A. Klatt and Hartmut Löwen.

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Keynote lecture	Anna Gusakova	14:30 – 15:20

Poisson hyperplane tessellation in constant curvature spaces

Anna Gusakova
(Münster University)

Consider a Poisson point process of hyperplanes (totally geodesic subspaces of dimension $d-1$) in d-dimensional space of constant sectional curvature κ. This countable random family of hyperplanes dissect the space into convex regions. Resulting construction leads to a tessellation called Poisson hyperplane tessellation. In Euclidean space, corresponding to $\kappa = 0$, the Poisson hyperplane tessellation is a classical model of stochastic geometry and numerous results about it's properties are available.

In this talk we study different characteristics of Poisson hyperplane tessellation, including cell intensities and expected Quermassintegrals of the zero cell, with a particular focus on their dependence on the curvature parameter κ.

Based on a joint work with Zakhar Kabluchko and Christoph Thäle.

Talk	Daniel Hug	15:20 – 16:00

Boolean Models in Hyperbolic Space

Daniel Hug
(Karlsruhe Institute of Technology)

The union of the particles of a stationary Poisson process of compact (convex) sets in Euclidean space $\mathbb{R}^d$ is called Boolean model and is a classical topic of stochastic geometry. We consider Boolean models in hyperbolic space $\mathbb{H}^d$, where one takes the union of the particles of a stationary Poisson process in the space of compact (convex) subsets of the hyperbolic space. Geometric functionals such as the volume of the intersection of the Boolean model with a compact convex observation window are studied. In particular, the asymptotic behavior for balls with increasing radii as observation windows is investigated. Exact and asymptotic formulas for expectation, variances and covariances are shown and univariate and multivariate central limit theorems are derived. Compared to the Euclidean framework, some new phenomena can be observed.

Based on joint work with Günter Last and Matthias Schulte.

Talk	Pierre Calka	16:30 – 17:10

Fractal random sets generated by iterated planar regular tessellations

Pierre Calka
(Université de Rouen Normandie)

We consider a regular tessellation of the plane, either square, triangular or hexagonal and we construct an increasing sequence of random sets obtained by the union of homothetic tiles from this tessellation. We show that the sequence converges to a random compact set and we calculate explicitly the box-dimension and Hausdorff dimension of that limiting set. The method relies on a precise coding of the construction by multitype branching random processes.

This is joint work with Yann Demichel.

Talk	Cedric Gommes	17:30 – 18:10

Time-dependent stochastic models for inelastic neutron scattering data analysis

Cedric Gommes
(University of Liège and Fonds National de la Recherche Scientifique)

Most experimental methods available for analyzing nanostructures are limited by stringent experimental conditions, and this is notably the case for electron microscopy. When it comes to analysing nanostructures in their natural environment - even as ordinary as water at room temperature - scattering of x-rays or neutrons is often the only option [1].

It is a well-known fact that the structural information obtained from *elastic* scattering experiments is equivalent to a covariance, and stochastic models have long been used to analyze this type of data. When it comes to studying systems that evolve in time, under the effect of thermal fluctuations, *inelastic* scattering experiments are possible. The structural information obtained from such experiments is a space- and time-dependent covariance $C(r, \tau)$, defined as the probability for two points at distance r from one another to belong to the same phase with a time lag τ [2]. Analyzing this type of data calls for novel time-dependent stochastic models.

In this presentation, we discuss time-dependent generalizations of two classical stochastic models, namely the clipped Gaussian field and the Boolean models. We illustrate how these models are used to jointly analyze Small-Angle Neutron Scattering (SANS) and Neutron Spin-Echo (NSE) data, and capture the space- and time-dependent structures of microemulsions [3] and of aerogels [4].

References

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Talk **Ilya Molchanov** **17:50 – 18:30**

Set-valued recursions arising from vantage-point trees

Ilya Molchanov
(University of Bern)

Vantage-point trees were introduced in the information processing literature in relation to nearest neighbour search problem in databases interpreted as metric spaces. The left-most path on such a tree is a sequence of points such that the next point lies in a suitably defined (decreasing) neighbourhood X_n of the previous one. The geometric rate of decrease is controlled by a parameter $\tau \in (0,1)$. After normalising $Y_n = \tau^{-1} X_n$, the random convex sets Y_n, $n \geq 1$, satisfy the following recursive equation

$$Y_{n+1} = \tau^{-1}(Y_n - U_{Y_n}) \cap K, \quad n \geq 1,$$

where U_{Y_n} is a point uniformly distributed in Y_n (conditionally on Y_n), and $Y_0 = K$ is a convex body K which contains the origin in its interior (usually, the ball in a certain metric).

We prove that $(Y_n)_{n\geq 0}$ is a recurrent Harris chain on the space of convex bodies in $\mathbb{R}^d$. Its stationary distribution is a generic ball polyhedron, that is, an a.s. finite intersection of independent copies of $(K - U_K)$ scaled by the powers of $\tau{-}1$. If K is a Euclidean ball, we obtain a ball polyhedron. As a consequence, we derive a limit theorem for the length of the leftmost path of a vantage-point tree.

Some other iterative schemes for random convex bodies will be also briefly discussed.

Joint work with C. Dong and A. Marynych.

Friday, 7 June 2024

09:00 – 09:50	**KN Miermont**	*Combinatorial and probabilistic aspects of maps*
09:50 – 10:30	**Sporring**	*Statistical investigations of random programs*

coffee break

11:00 – 11:40	**d'Alayer**	*Sequential Thinning subsampling method for learning Large-Size Latent-Marked Point Processes*
11:40 – 12:20	**Bille**	*Random eigenvalues of graphene and the triangulation of plane*

lunch

Keynote lecture **Grégory Miermont** **09:00 – 09:50**

Combinatorial and probabilistic aspects of maps

Grégory Miermont
(École Normale Supérieure de Lyon)

A map is a topological surface obtained by identifying in pairs the edges of a finite collection of polygons. This gives rise, for instance, to the familiar concept of a triangulation of the sphere, which is widely used in 3D computer imaging, however, in our situation, we consider maps as abstract combinatorial objects rather than discretization of surfaces embedded in the space. With this point of view, the countable collection of maps of a given topology can be seen as a set of discrete metrics defined on a fixed topological surface, for instance, by endowing the set of vertices of a map with the combinatorial graph distance. Therefore, considering models of random maps, for instance, a uniformly random triangulation of the sphere with a fixed number 2n of triangles, provides a natural setting for studying random metrics. In this context, a question of interest concerns the asymptotic geometry of such a random object as n goes to infinity.

In this talk, we will review some of the relevant combinatorial aspects of maps, and discuss how it allows to define limiting objects that are, in a sense, canonical models of random surfaces.

Talk **Jon Sporring** **09:50 – 10:30**

Statistical investigations of random programs

Jon Sporring
(University of Copenhagen)

We consider all the grammatically correct programs of a given programming language as a metric space (M, d), where M is the discrete set of all possible programs and $d : M \times M \to \mathbb{R}$ is a distance function between them. The set M is countable infinite and embodies all programs, while the distance function d describes some notion of similarity between programs. Given a distance function, we can now construct a fully connected graph $G = (M, E)$ with programs being vertices M and edges E between programs that have the distance as an attribute. For practical reasons, we will consider subsets $M' \subset M$ and their distances $d_{ij} = d(m_i, m_j)$, $m_i, m_j \in M'$. With the set $\{d_{ij}\}_{i<j}$, we can apply a suite of standard tools for describing the geometry, topology, and statistics of M' as induced by the choice of d. For example, if M' is a random subset of M, then we may calculate its geometric medians and absolute distance [2], its density and tendency to cluster [4], and its persistent homology diagrams and their statistics [1].

The applicability of the suite of tools we present here is independent of the choice of d, but the ensuing analysis is not. Here, we favor tree-edit distances [5] of parsed grammar trees since the parse-tree is a concise description of the syntax of a program, highlighting programming constructs rather than the spelling of keywords, etc. Further, we focus on Python programs due to their widespread appeal for small programs written by non-computer science and with the aid of large language models [3]. Our setup opens up for performing semantically invariant transform of code before generating trees, nevertheless, here we limit ourselves to demonstrating how a suite of geometry, topology, and statistical tools for analyzing point sets can be used to study programming languages and stochastic program sources.

Joint work with Ken Friis Larsen.

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Talk	**François d'Alayer**	**11:00 – 11:40**

Sequential Thinning subsampling method for learning Large-Size Latent-Marked Point Processes

François d'Alayer de Costemore d'Arc
(INRAE BioSP)

In numerous fields, understanding the characteristics of a large-size marked point process (called the ground process) is crucial, particularly when the marks are latent and can only be inferred through sampling. In this aim, we need to gather an efficient sample which is a thinning of the ground process. To address this challenge, we introduce a sequential thinning approach tailored for diverse goals such as estimating the mark distribution. For each objective, a loss function is defined on the space of thinning of the ground process. Since this loss relies on unknown elements like the marks or the ground process, it remains unknown itself. We minimize this loss by sequentially minimizing an estimate of it. This involves coupling two distinct sequential thinning processes: initially employing a uniform sampling for error estimation, followed by a non-uniform sampling aimed at minimizing this estimated error. We illustrate our approach through a case study in plant disease surveillance. Here, the point process denotes the spatial locations of plants, with the binary mark indicating infection status. Our method facilitates the estimation of spatial mark distribution and the delineation of infected and non-infected regions.

Joint work with Samuel Soubeyrand and Edith Gabriel.

Talk	Artur Bille	11:40 – 12:20

Random eigenvalues of graphene and the triangulation of plane

Artur Bille
(Ulm University)

In 2010, Sir Konstantin Novoselov and Sir Andre Geim were awarded the Nobel price in Physics for their pioneering method to isolate single layers of *graphene*. Graphene is a carbon allotrope where the atoms are arranged in a cut-out of an infinitely large hexagonal lattice. It can be viewed as an extremal case of other finite-sized carbon allotropes like *fullerenes*.

From the mathematical perspective, *fullerenes* are planar, 3–regular graphs with only pentagonal and hexagonal facets. It is well–known that fullerenes always have twelve pentagons, with the number of hexagons varying. This amount is a linear function of the number of vertices n. As n approaches infinity, the increasing number of hexagons suggests that an infinitely large fullerene could resemble the previously mentioned hexagonal lattice.

In this talk, we discuss spectral properties of the hexagonal lattice and its dual, and we analyze the counts of closed paths of length $k \in \mathbb{N}$ on these lattices. These counts form a moment sequence of specific random variables related to the distance of a position of a planar random flight (after three steps) from the origin. Here, we refer to such a random variable as a *random eigenvalue* of the underlying lattice. Explicit formulas for the probability density and characteristic functions of these random eigenvalues are given for both the hexagonal and the triangular lattice. Furthermore, we demonstrate that both probability distributions can be approximated by a functional of the random variable uniformly distributed on expanding intervals $[0, b]$ as $b \to \infty$. This yields a straightforward method for simulating these random eigenvalues without generating graphene and triangular lattice graphs.

In the proof of this approximation, a novel integral identity for a specific series containing the third powers of the modified Bessel functions I_n of nth order, $n \in \mathbb{Z}$, is shown. Such series play a crucial role in various contexts, in particular, in analysis, combinatorics, and theoretical physics.

Joint work with Victor Buchstaber, Simone Coste, Satoshi Kuriki, and Evgeny Spodarev.

Poster Contributions

There is a dedicated **Poster Presentation Session** on Monday before dinner starting at 17:40 h, where each poster is introduced in a 2 minutes short presentation. After dinner it is continued with a **Poster Session**, which allows for individual and more in-depth discussions.

Poster 1 **Christian Bair**

Uniqueness of time-dependent density-potential mappings

Christian Bair
(Heinrich-Heine-Universität Düsseldorf)

One central question in liquid-state theory is whether the one-body density uniquely specifies the external potential. In equilibrium, such a mapping from the time-independent density to the potential has been established by Mermin [1]. Later, Chayes, Chayes, and Lieb have proven that this unique mapping exists under mild and natural conditions that depend only on the classical density and interparticle interactions [2]. This rigorous statement is now the cornerstone of classical density functional theory (DFT) [3] since it implies that knowing the one-body density means that, in principle, all information on the equilibrium system is available. Out of equilibrium, the conditions for a unique time-dependent density-potential mapping become much more intricate because of the tremendously larger state space.

Here, we derive rigorous and explicit conditions for a unique mapping between a nonequilibrium density profile and a time-dependent external potential. We thus prove the underlying assertion of dynamical density functional theory (DDFT) — with or without the so-called adiabatic approximation often used in applications [4, 5]. We also illustrate loopholes when our conditions are violated so that two distinct external potentials result in the same density profiles but different currents, as suggested by the framework of power functional theory (PFT) [6].

This is a joint work with Michael A. Klatt, Hartmut Löwen and René Wittmann.

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Poster 2 **Dominik Beck**

On Random Simplex Picking

Dominik Beck
(Charles University)

Selected values of odd random simplex volumetric moments are derived in an exact form in various bodies in dimensions three, four, five and six. In three dimensions, the well known Efron's formula is also used. As it turned out, the problem is solvable in higher dimensions too using nothing more than Blashke-Petkanchin formula in Cartesian parametrisation in the form of the Canonical Section Integral.

Acknowledgements

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Poster 3 **Tillmann Bühler**

Intersection processes of k-flats in hyperbolic space: New limits and convergence rates for observations in spherical windows

Tillmann Bühler
(Karlsruhe Institute of Technology)

Over the past years, there has been a lot of interest in the study of isometry invariant Poisson processes of k-planes in hyperbolic space $\mathbb{H}^d$ $(0 \leq k < d)$. A phenomenon that has no counterpart in euclidean geometry arises in the investigation of the total k-dimensional volume F_r of the process inside a spherical observation window B_r, when one lets r tend to infinity: While F_r is asymptotically normally distributed for $2k \leq d+1$, it has been shown to obey a nonstandard central limit theorem for $2k > d+1$.

The intersection process of order m (for $d-m(d-k) \geq 0$) of the original process η consists of all intersections of distinct planes $E_1, \dots, E_m \in \eta$ with $\dim(E_1 \cap \dots \cap E_m) = d - m(d-k)$. For this process, the total $d-m(d-k)$-dimensional volume $F_r^{(m)}$ of the process in B_r is of particular interest. For $2k \leq d+1$ it has been shown that $F_r^{(m)}$ is again asymptotically normally distributed. For $m \geq 2$, the limit is so far unknown, although it has been shown for certain d and k that it cannot be a normal distribution. We determine the limit distribution for all values of d, k, m. We also give explicit rates of convergence in the Kolmogorov distance.

Joint work with Daniel Hug.

Poster 4 **Mikhail Chebunin**

Uniqueness of the infinite cluster in the random connection model

Mikhail Chebunin
(Karlsruhe Institute of Technology)

We consider a random connection model (RCM) on a general space driven by a Poisson process. We say that the infinite clusters are deletion stable if the removal of a Poisson point cannot split a cluster in two or more infinite clusters. We will show that this stability together with a natural irreducibility assumption implies uniqueness of the infinite cluster. Conversely, if the infinite cluster is unique then this stability property holds. We will also show that the stationary marked RCM, which encompasses the Boolean model with general compact grains and the so-called weighted RCM as special cases, satisfies the deletion stability property.

Joint work with Günter Last.

Poster 5 **Fabio Frommer**

An inverse cluster expansion for the chemical potential

Fabio Frommer
(Universität Mainz)

Interacting particle systems in a finite volume in equilibrium are often described by a grand canonical ensemble induced by the corresponding Hamiltonian, i.e. a finite volume Gibbs measure. However, in practice, directly measuring this Hamiltonian is not possible, as such, methods need to be developed to calculate the Hamiltonian potentials from measurable data. In this work we give an expansion of the chemical potential in terms of the correlation functions of such a system in the thermodynamic limit. This is a justification of a formal approach of Nettleton and Green from the 50's.

Poster 6 **Kateřina Helisová**

Assessing dissimilarity of random sets based on their geometric characteristics

Kateřina Helisová
(Czech Technical University in Prague)

The contribution concerns a statistical method for assessing dissimilarity of two realisations of random sets. The method focuses on shapes of the components of the realisations, namely on the curvature of their boundaries together with the ratios of their perimeters and areas. First, theoretical background is introduced. Then, the method is described, justified by a simulation study and applied to real data of two different types of tissue - mammary cancer and mastopathy.

Joint work with Vesna Gotovac Đogaš, Bogdan Radović and Jakub Staněk.

References

Gotovac Đogaš V., Helisová K., Radović B., Staněk J., Zikmundová M., and Brejchová K. (2021): Two-step method for assessing similarity of random sets. *Image Analysis and Stereology*, **40**, 127–140.

Poster 7 **Frankie Higgs**

Connecting two points with random blobs

Frankie Higgs
(University of Bath)

The *random blob* model, or the Boolean model, places balls of radius r centred at the points of a homogeneous Poisson point process $\mathcal{P}$ in a set A with an intensity μ. It is closely related to the random geometric graph, in which two points of $\mathcal{P}$ are joined with an edge if the distance between them is less than $2r$. If the relationship between the number of balls and the radii is such that the average degree of a point, which is proportional to μr^d, converges to a constant then the model can be thought of as a form of percolation.

Indeed, if $A = \mathbb{R}^d$ or a half-space and $r = 1$, the random blob model is often called *continuum percolation*, parameterised by the intensity. Many well-known properties of discrete percolation are known to also hold for continuum percolation, such as existence of a non-trivial critical intensity and the existence of a unique unbounded component.

We will discuss recent work on the two-point connection function for the Boolean model. If A is a bounded subset of $\mathbb{R}^d$ with a sufficiently smooth boundary, then given distinct $x, y \in \partial A$, we prove a limit theorem for the probability that x and y are joined by a path in the Boolean model. In particular, if $\theta_A(\lambda)$ is the probability that the origin is contained in an unbounded component in continuum percolation with intensity λ in A, then we show that if $\lambda r^d \to \mu$ as $\lambda \to \infty$, then the probability that x and y lie in the same component converges to $\theta_{\mathbb{H}}(\lambda)^2$.

Our proof involves a renormalisation argument relating certain events for the Boolean model in A explicitly to corresponding events for continuum percolation in $\mathbb{H}$.

Based on joint work with Mathew Penrose.

Poster 8 **Mathias in Wolde-Lübke**

Random Laguerre tessellations

Mathias in Wolde-Lübke
(University of Münster)

In this poster we present a family of random Laguerre tessellations $\mathcal{L}_d(f)$ in $\mathbb{R}^d$ as well as their duals, generated by inhomogeneous Poisson point processes in $\mathbb{R}^d \times \mathbb{R}$ whose intensity measures have density of the form $(v, h) \mapsto \gamma f(h)$ under some natural restrictions on the functions f. We show that the construction provides a tessellation and establish a connection to fractional calculus. This family includes the models introduced in [1] and [2] as well as random Laguerre tessellations constructed by independently marked homogeneous Poisson point processes in $\mathbb{R}^d$ studied in [3]. We consider the intersections of the random Laguerre tessellation $\mathcal{L}_d(f)$ with an affine ℓ-dimensional subspace and show that this leads to an ℓ-dimensional random Laguerre tessellation $\mathcal{L}_\ell(\widetilde{f})$, where $\widetilde{f}$ is a fractional integral of order $\frac{d-\ell}{2}$ of f. Further the distribution of the volume weighted typical cell of the corresponding dual tessellation is explicitly given in terms of f.

Based on joint work with Anna Gusakova.

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Poster 9 **Bochen Jin**

Convergence of High Dimensional Random Walk

Bochen Jin
(University of Bern)

It is shown in [1] that the metric space derived from the first n steps of a random walk in the d-dimensional Euclidean space converges in probability to the Wiener spiral under the Gromov-Hausdorff distance as the number of steps n and the dimension d increase to infinity. The Wiener spiral is isometric to $[0,1]$ with the metric $r(t,s) = \sqrt{|t-s|}$.

By a similar approach and under the same conditions, one of our results states that the transformes random walk converges to the bridge variant of the Wiener spiral, which is isometric to $[0,1]$ with the pseudometric $\sqrt{|t-s|(1-|t-s|)}$.

We also extend the results of [1] for ℓ_p metrics. The main step is to prove that the ℓ_p norm of d-dimensional random walk with n steps which have i.i.d. components or exchangeable components, converges uniformly to $c(p)\ t^{p/2}$ in probability as $n, d \to \infty$ with constant $c(p)$ depending on p. The limit of the path equipped with the ℓ_p metrics is also the Wiener spiral.

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ter

Poster 10 **Matthias Lienau**

Large components in the subcritical random connection model

Matthias Lienau
(Hamburg University of Technology)

The random connection model is an inhomogeneous random graph whose vertex set is given by a stationary Poisson process in $\mathbb{R}^d$. All vertices are equipped with positive i.i.d. weights. Any two vertices get connected via an edge with a probability which is increasing in the weights of the endpoints but decreasing in their distance. The resulting graph exhibits natural clustering effects as well as a scale-free behaviour for a suitable weight distribution. For our results we require the graph to have sufficiently few edges. We study the point process of component sizes in a growing observation window and derive, under suitable rescaling, weak convergence to a Poisson process. This implies that the rescaled size of the largest component in the observation window converges to a Fréchet distribution.

This talk is based on joint work with Matthias Schulte.

Poster 11 **Luca Lotz**

Constructing Hyperuniform Thinnings of α-Mixing Point Processes through Stable Matching

Luca Lotz
(Karlsruhe Institute of Technology and German Aerospace Center)

Hyperuniformity is a global property of stationary point processes. The variance of the number of points of such processes within an observation window grows substantially more slowly than the volume of the observation window. At a macroscopic level, this results in rather uniformly distributed points. This distinguishes hyperuniform point processes from, for example, Poisson processes, where points are not correlated, and the variance is proportional to the volume of the observation window.

We explore the construction of hyperuniform thinnings of a broad class of α-mixing stationary point processes. For a small thinning parameter, the thinned point process is locally hardly distinguishable from the original point process. The construction is based on stable matching, which describes a specific pairwise matching of points from two point processes, favoring points in close proximity. The concept is known from the ßtable marriage problem", which in this context is uniquely solvable by a simple algorithm. Here the target point process is stably matched with a hyperuniform point process of lower intensity. Upon disregarding points of the target process that lack a matching partner, the resulting thinning inherits hyperuniformity. This work extends a prior result in [1], where a more limited class of point processes was considered.

The findings originate from my master's thesis, which was supervised by Günter Last.

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Pos-
ter

Poster 12 **Marcela Mandarić**

Topological data analysis for random sets and its application in detecting outliers and goodness of fit testing

Marcela Mandarić
(University of Split)

We present the methodology for detecting outliers and testing the goodness-of-fit of random sets using topological data analysis. We construct the filtration from level sets of the signed distance function and consider various summary functions of the persistence diagram derived from the obtained persistence homology such as accumulative persistence function and support function of lift zonoid. The outliers are detected using functional depths for the summary functions. Global envelope tests using the summary statistics as test statistics were used to construct the goodness-of-fit test. The procedures were justified by a simulation study using germ-grain random set models.

Joint work with Vesna Gotovac Đogaš.

Poster 13 **Nikolaj Nyvold Lundbye**

Poisson approximation of extreme topological features

Nikolaj Nyvold Lundbye
(Aarhus University)

In topological data analysis, features of a large lifetime are of central importance as they provide a hint on interesting structures hidden in the dataset. In a seminal work, O. Bobrowski, M. Kahle and P. Skraba determined the order of the maximal lifetime in the Čech complex on a stationary Poisson point process in the unit cube as the intensity tends to infinity. However, so far there is no mathematically rigorous analysis of more refined asymptotic distributional properties.

In this talk, I will explain how to establish such distributional properties when the deathtime of features is bounded by a connectivity radius where 1) the expected number of neighbors of a typical point tends to 0, and 2) the large-lifetime-features appear with high probability in clusters of a certain size. More precisely, I will explain how to establish a Poisson convergence result for the location of the large-lifetime-features which can be further refined to also include the feature's deviation to the maximal lifetime and maximal deathtime. If time permits it, we will also see what can be done when the cluster size grows to infinity - that is with no bound on the deathtime.

The talk is based on joint work with Christian Pascal Hirsch and Moritz Otto.

Poster 14 **Zbyněk Pawlas**

Applications of random marked tessellations

Zbyněk Pawlas
(Charles University)

A random marked tessellation is obtained by attaching a random mark to each cell within a random tessellation. In the stationary case, we consider estimators of the typical mark distribution and examine their asymptotic behaviour as the observation window expands. We demonstrate applications of this model motivated by examples from the study of polycrystalline materials. Of particular interest are the marks lying in some non-Euclidean space. Such marks may represent, for example, crystallographic orientations or stress tensors.

Poster 15 **Martina Petráková**

Some notes about Poisson–Laguerre tessellation with unbounded weights

Martina Petráková
(Charles University)

The object of our research is the Poisson–Laguerre tessellation, i. e. a random Laguerre tessellation whose generator is a Poisson marked point process with intensity t. We are interested in the asymptotic behaviour (as $t \to \infty$) of functionals of the tessellation – e. g. the perimeter of the cells, the ratio of volumes of the neighbouring cells – in the case where the weights of the random generator are not uniformly bounded. As it turns out, it is useful to study the behaviour of the distance to the furthest neighbour of a typical point of the point process. In this contribution we will present some properties of this characteristic, which were derived using the concept of tempered configurations.

Joint work with Zbyněk Pawlas.

Poster 16 **Bogdan Radović**

Classification of Realisations of Random Sets

Bogdan Radović
(Czech Technical University in Prague)

This contribution concerns methods for classification of realisations of random sets. The methods combine functional data analysis and spatial statistics procedures derived for random sets. We focus on functional characteristics evaluated from individual components in the realisations based on their shapes. The functional data obtained in such a way is then used for nonparametric classification using both supervised and unsupervised approach. The proposed methods have been justified through a simulation study and applied to real medical data.

Joint work with Kateřina Helisová.

References

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Poster 17 **Mathias Sonnleitner**

Gaps in random inscribed polytopes

Mathias Sonnleitner
(University of Münster)

The convex hull K_n of n independently and uniformly distributed points on the surface of a convex body K is a well studied model of a random polytope. In particular, there are asymptotic results on the distance between K and K_n or the combinatorial structure of K_n. We survey existing results, point to gaps in the literature and present (partial) progress towards closing them. More precisely, we study asymptotics for the expected Hausdorff distance between K_n and K when K is itself a polytope. For smoothly bounded K we study fluctuations for the Hausdorff distance between K_n and K, which come in the shape of extreme value distributions, as well as for the number of faces of K_n for which a central limit theorem holds.

Poster 18 **Chenhao Wang**

3D Shape Analysis and Bias Correction with Persistent Homology

Chenhao Wang
(University of Copenhagen and Center for Quantification of Imaging Data from MAX IV)

Accurate characterization of 3D shapes is central in many research fields and various methods have been proposed for measuring one of the most popular 3D shape parameters: object thickness. Examples include: local thickness [3], based on fitting the largest spheres possible throughout the object of interest and labeling the thickness at each voxel with the largest radius covering that point; M-reps [2], where the thickness is computed from the object's medial means; and our persistent homology method proposed in a recent paper [1]. In the latter, image dilation/erosion is applied as filtration to produce curves recording the number of holes/connected components at any given time step. From these, the object thickness and roughness are extracted as the index of maximum count and the full width at half maximum, respectively.

In this study, we perform empirical investigations into the accuracy of the mentioned methods by applying them to synthetically generated objects with well-defined theoretical thicknesses. The objects cover a wide variety of different shapes, thicknesses, and edge roughnesses, and the experiments are repeated thousands of times to ensure statistical reliability. In general, we find that the methods tend to underestimate the thickness (defined as the difference between two central edges), and consequently, a bias compensator was implemented to correct this by fitting a function that models the negative difference in the measured thickness using our roughness measure.

Through this study, we hope to provide a more accurate thickness measurement method that is validated across a variety of different shapes.

Joint work with Stine Hasselholt and Jon Sporring.

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[2] SM. Pizer *et al.*, Deformable M-Reps for 3D Medical Image Segmentation, *International Journal of Computer Vision*, vol. 55, no. 2, 2003.

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